REFERENCE RATE METHODOLOGIES OF CRYPTOCURRENCIES

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Abstract. We propose a real time benchmark for bitcoin called QRTR whose calculation is much simpler than that of BRTI. Then, we introduce two types of reference rates that are usable for daily quotes of the currency, based on QRTR. The methodologies we propose in this paper are applicable not only for bitcoins but also for other cryptocurrencies.

1. Introduction

As bitcoin and other cryptocurrencies have been widely traded, their fair prices are strongly demanded for their daily settlements. There are many exchanges or clearing houses handling cryptocurrencies, and each of them has their own quotes. Then, there are two problems in their handling of cryptocurrencies. One problem is that the mid prices of exchanges are very different each other, the other problem is that their bid-ask spreads are usually quite big. These two problems make it difficult to provide a reliable fair price to investors.

In order to solve the problems, CME gives two benchmarks for bitcoin, called BRTI (Bitcoin Real Time Index) and BRR (Bitcoin Reference Rate). They are launched in 2016 and are widely used in United States. However, the BRTI methodology is somehow complex and not so robust toward the time dimension.

In this paper, we propose another index or benchmark methodology for bitcoin, called QRTR(QUICK Real Time Rate) whose computation is simpler and faster than that of BRTI and more robust toward the time dimension.

We also propose a reference rates based on QRTR that are corresponding to BRR.

2. The BRTI methodology

The BRTI (Bitcoin Real Time Index) methodology was developed by Crypto Facilities Ltd. [Crypt, 2017a]. It is a US Dollar based index of bitcoin, and has been used in CME since 2016.

In this section, We give a precise mathematical definition for it, and mention its pros and cons.

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The BRTI at time t is calculated with a consolidated limit order book (LOB) L_t at t. It is a cross-sectional value and does not depend on the past data.

The LOB L_t consists of an ask part and a bid part like $L_t := (A_t, B_t)$, where

$$A_t := \{(p^A_{t,i}, s^A_{t,i})\}_{i=1,2,\ldots,N}, \quad B_t := \{(p^B_{t,i}, s^B_{t,i})\}_{i=1,2,\ldots,N}, \tag{2.1}$$

the sequence of pairs of prices and shares, and N is a sufficiently large number for holding all LOB data during the period we are interested in. We assume that all prices and shares have positive values, and that for every i and j such that $i < j$,

$$p^A_{t,i} < p^A_{t,j}, \quad p^B_{t,i} > p^B_{t,j}, \quad p^A_{t,1} > p^B_{t,1}. \tag{2.2}$$

In this section, the character 'Z' stands for 'A' or 'B', specifying 'ask' or 'bid'.

Let C be a positive constant value specifying an order size cap (say, $C := 100$). For $i = 1, 2, \ldots, N$, define

$$\dot{s}^Z_{t,i} := \min\left(s^Z_{t,i}, C\right), \quad \dot{Z}_t := \{(p^Z_{t,i}, \dot{s}^Z_{t,i})\}_{i=1,2,\ldots,N}, \quad \dot{L}_t := (\dot{A}_t, \dot{B}_t). \tag{2.3}$$

Now for $v \leq 0$, define two functions ℓ^A_t and ℓ^B_t by

$$\ell^Z_t(v) := \sup\left\{\ell \mid \sum_{i=1}^{\ell} \dot{s}^Z_{t,i} < v\right\} + 1. \tag{2.4}$$

We use a convention $\sup\emptyset = 0$. Hence $\ell^Z_t(0) = 1$. It is also obvious that ℓ^Z_t is an increasing function, that is, $v < v'$ implies $\ell^Z_t(v) \leq \ell^Z_t(v')$.

For $v \leq 0$, we define the following values.

$$p^Z_t(v) := p^Z_{t,\ell^Z(v)}, \tag{2.5}$$

$$p^M_t(v) := \frac{1}{2}(p^A_t(v) + p^B_t(v)), \tag{2.6}$$

$$p^S_t(v) := \frac{p^A_t(v) - p^B_t(v)}{p^A_t(v) + p^B_t(v)} = \frac{p^A_t(v)}{p^M_t(v)} - 1 = 1 - \frac{p^B_t(v)}{p^M_t(v)}. \tag{2.7}$$

Then, we have the following proposition.

Proposition 2.1. *Suppose that $v < v'$. Then, we have:*

$$p^A_t(v) \leq p^A_t(v'), \tag{2.8}$$

$$p^B_t(v) \geq p^B_t(v'), \tag{2.9}$$

$$p^S_t(v) \leq p^S_t(v'). \tag{2.10}$$

Proof. (2.8) and (2.9) are obvious since ℓ^Z_t is monotonic.

Let us define a function f by

$$f(x, y) := \frac{x - y}{x + y} \quad (x, y > 0). \tag{2.11}$$

Then

$$\frac{\partial f}{\partial x}(x,y) = \frac{2y}{(x+y)^2} > 0, \quad \frac{\partial f}{\partial y}(x,y) = -\frac{2x}{(x+y)^2} < 0. \tag{2.12}$$

Therefore, by (2.8) and (2.9), we obtain (2.10). □

Finally, we define BRTI_t by

$$\mathrm{BRTI}_t := \Big(\int_0^{\bar{v}_t} p_t^M(v) e^{-\lambda_t v} dv\Big) / \Big(\int_0^{\bar{v}_t} e^{-\lambda_t v} dv\Big), \tag{2.13}$$

where

$$\bar{v}_t := \sup\{v \geq 0 \mid p_t^S(v) \leq D\}, \tag{2.14}$$

λ_t and D are positive numbers like

$$\lambda_t := \frac{1}{0.3\bar{v}_t}, \quad D := 0.005. \tag{2.15}$$

Note that $\bar{v}_t$ is well-defined by (2.10) in a sense that once we have $p_t^S(v_0) > D$, we will never have $p_t^S(v) \leq D$ for $v > v_0$.

Since the BRTI is defined by a cross-sectional data, it is robust in the price dimension or the space dimension. However, it can not detect the anomaly in the time dimension since it does not depend on the past information, which means it can pick temporal outliers that may not adequate to use as a reference price as the ingredients of its calculation.

3. The QRTR methodology

In this section, we will introduce a Japanese Yen based benchmark for bitcoin called QRTR(tentative)(QUICK Real Time Rate).

The QRTR(tentative) is calculated with a time series of executions

$$\{X_i\}_{i=1,2,\dots} \tag{3.1}$$

in which X_i is the i-th execution of the form

$$X_i := (t_i, p_i, s_i), \tag{3.2}$$

where t_i, p_i and s_i are transaction time, Japanese YEN based price and shares of the execution, respectively. We, of course, assume that $t_i < t_j$ for every pair of i and j with $i < j$.

Now we define the QUICK Real Time Rate or QRTR at time t by the following equation.

$$\mathrm{QRTR}_t := \Big(\sum_{t_i \leq t} p_i s_i e^{-\lambda(t-t_i)}\Big) / \Big(\sum_{t_i \leq t} s_i e^{-\lambda(t-t_i)}\Big). \tag{3.3}$$

The definition of QRTR is much simpler than that of BRTI. The Figure 3.1 shows some 6 days QRTR values compared with the corresponding raw prices that are taken from transaction data at bitFlyer[1].

[1]https://bitflyer.com/en/

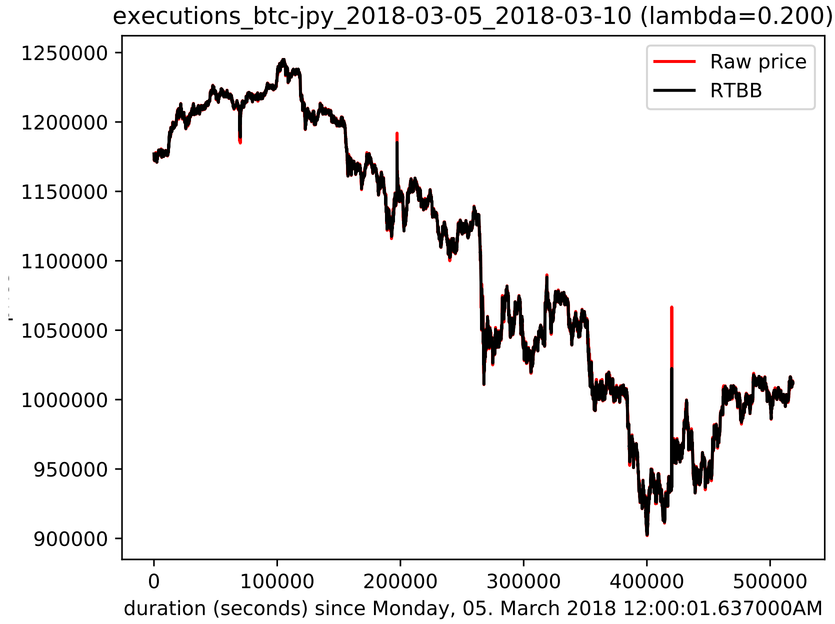

Figure 3.1. QRTR

4. Analysis of QRTR

In this section, we discuss the robustness of QRTR, especially by focusing on how stable it is against intentional and/or unintentional outliers.

At a first glance of Figure 3.1, you will see that the QRTR values (black line) are less volatile than raw prices (red line).

Figure 4.1 is a zoom shot of the most volatile period in Figure 3.1.

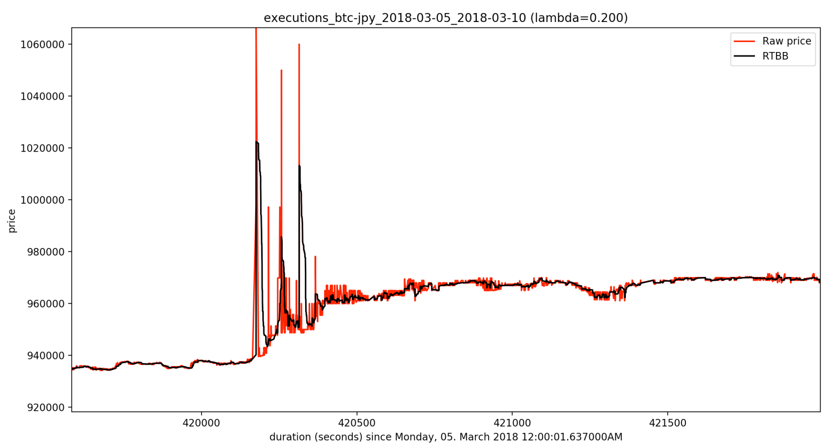

Figure 4.1. QRTR(zoomed up)

In (3.3), two summentions are made over all data from the time negative infinity through the current time t. But, what if we only see the data after some limited past time, say one day ago, in other words when we use the limited windows for the calculation.

Against the data of two whole days of March 5th through 6th of the year 2018, we calculated the maximum difference of the two calculation methods, using the data from the time negative infinity and using the data after s seconds past time for two cases $s = 60$ (one minute) and $s = 3600$ (one hour) when $\lambda = 0.2$. They are 9.5025 JPY for one minute, and $4.4238e - 09$ JPY for one hour. The latter is completely negligible small. Moreover the computation times for limited windows are significantly big. See Table 4.1[2]. Therefore, thinking situations when we pick a window whose length is more than one hour, we can use the "from-the-time-negative-infinity" method that is very much faster than using the limited windows.

window size (sec)	maximum diff (JPY)	computation time (sec)
∞	0	3.738
3600	$4.4238e - 09$	1605.373
60	9.5025	40.920

Table 4.1. Maximum differences of QRTR per window sizes on 2018-03-05 through 2018-03-06

5. Reference Rates based on QRTR

In this section, we introduce two methods for providing daily quotes or reference rates of bitcoin. Both are calculated based on QRTR.

We assume that we already have a QRTR stream, say $\{\mathrm{QRTR}_t\}_{t\in\mathcal{T}}$, where $\mathcal{T}$ is a given time domain.

Our goal is to provide robust reference rates against intentional outliers near quoting time that are set at the same time t_0 every day, say 4pm. In order to make it possible we see prices in a window containing the quoting time t_0 instead of focusing on the single price at t_0 since we think the price at t_0 may be contaminated by intentional manipulations done by market participants.

The window is determined as a closed interval

$$W_c := [t_0 - c, t_0 + c] \tag{5.1}$$

with a pre-defined small positive constant c. Then, we obtain a subsequence of QRTR values

$$\mathrm{QRTR}_c := \{\mathrm{QRTR}_t\}_{t\in W_c} \tag{5.2}$$

[2]Using Python 3.6.3 and macOS 10.13.6 running on iMac (4.2GHz Intel Core i7, 64GB RAM).

Now, we pick a reference rate QCRR(tentative) (QUICK Cryptocurrency Reference Rate) by the following algorithm.

(1) $c := c_0$ where c_0 is a predefined number, say 30 seconds.
(2) $n := |\mathrm{QRTR}_c|$, the number of prices containing in QRTR_c.
(3) if n equals to 0, then make $c := c * 2$ and go back to (2).
(4) $\mathrm{QCRR} := \varphi(\mathrm{QRTR}_c)$.

The function φ is a *picking function*. We propose two types of picking functions φ_{RND} and φ_{MED}.

- φ_{RND}
  Pick a QRTR value from QRTR_c by uniformly random way.
- φ_{MED}
  Sort values in QRTR_c and get a median value from the resulting list.

In the following table, we have reference rates

$$\varphi_{RND}(\mathrm{QRTR}_{30\mathrm{sec}}) = 1013983.592$$
$$\varphi_{MED}(\mathrm{QRTR}_{30\mathrm{sec}}) = 1013027.050$$

on March 10, 2018.

t_i	s_i	p_i	QRTR_{t_i}	$\mathrm{QRTR}_{t_i} - p_i$
15:59:33.757000	0.1490000000	1013036.000	1013026.654	-9.346
15:59:35.417000	0.0088000000	1013027.000	1013026.663	-0.337
15:59:37.627000	0.0197000000	1013021.000	1013026.222	5.222
15:59:40.660000	0.0500000000	1012999.000	1013018.983	19.983
15:59:41.270000	0.0050000000	1012989.000	1013018.109	29.109
15:59:41.833000	0.0957250000	1012989.000	1013006.914	17.914
15:59:44.943000	0.0100000000	1013017.000	1013007.616	-9.384
15:59:47.657000	0.2900000000	1013017.000	1013014.903	-2.097
15:59:47.657000	0.2803000000	1013021.000	1013017.517	-3.483
15:59:47.657000	0.1000000000	1013027.000	1013018.775	-8.225
15:59:49.317000	0.5000000000	1013036.000	1013027.050	-8.950
16:00:02.383000	0.0435112900	1013118.000	1013060.082	-57.918
16:00:04.573000	0.0050000000	1013118.000	1013063.600	-54.400
16:00:13.710000	0.0010000000	1013609.000	1013101.905	-507.095
16:00:16.950000	0.0010000000	1013622.000	1013163.470	-458.530
16:00:22.737000	0.0010000000	1013980.000	1013386.859	-593.141
16:00:26.710000	0.0040000000	1013980.000	1013806.686	-173.314
16:00:26.710000	0.3073000000	1013984.000	1013980.798	-3.202
16:00:26.710000	0.2500000000	1013984.000	1013982.220	-1.780
16:00:26.710000	0.3540000000	1013985.000	1013983.293	-1.707
16:00:26.710000	0.1947000000	1013985.000	1013983.592	-1.408
16:00:26.710000	φ_{RND}	1013985.000	1013983.592	-1.408
	φ_{MED}	1013036.000	1013027.050	-8.950

TABLE 5.1. Executions in $W_{30\mathrm{sec}}$ on 2018-03-10

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